



A review of numerical fluids/structures interface methods for computations using high-fidelity equations

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Abstract

Domain decomposition approaches require efficient interface techniques when fluids and structures are solved in independent computational domains for aerospace applications. Fluid/structure interfacing techniques for solutions from equations based on low-fidelity approaches that are in the linear domain are well advanced and are incorporated in production codes NASTRAN and ASTROS. However, for computations involving high-fidelity equations such as the Navier–Stokes for fluids and finite elements for structures, interface approaches are still under development. This paper provides a technical overview of methods for interfacing flow solutions from the Euler/Navier–Stokes methods with structural solutions using modal/finite-element methods. Validity of the methods is supported by previously presented results. Published by Elsevier Science Ltd.

Keywords: Finite elements; Navier–Stokes; Computers; High fidelity; Aeroelasticity

1. Introduction

Aeroelasticity that involves strong coupling of fluids and structures is an important element in the design of aircraft. Methods to couple fluids and structures by using low-fidelity methods such as the linear aerodynamic flow equations coupled with the modal structural equations are well advanced. Although these low-fidelity approaches are used for preliminary design, they are not adequate for the analysis of aircraft which can experience complex flow/structure interactions that require the use of high-fidelity approaches. Supersonic transports can experience vortex-induced aeroelastic oscillations [1] and subsonic transports can experience transonic buffet associated structural oscillations [2]. Both may experience a dip in flutter speed in the transonic regime. High-fidelity equations such as the Euler/Navier–Stokes (ENS) for fluids directly coupled with finite elements

(FEs) for structures are needed for accurate aeroelastic computations for which these complex fluid/structure interactions exist. Using these coupled methods, design quantities such as structural stresses can be directly computed. Using high-fidelity equations involves additional complexities from numerics such as higher-order terms. Therefore the coupling process is more elaborate when using high-fidelity methods than it is for calculations using linear methods.

In recent years, significant advances have been made for single disciplines in both computational fluid dynamics (CFD) using finite-difference (FD) approaches [3] and computational structural mechanics (CSM) using finite-element methods (see Chapter I of Ref. [4]). For aerospace vehicles, structures are dominated by internal discontinuous members such as spars, ribs, panels, and bulkheads. The FE method, which is fundamentally based on discretization along physical boundaries of different structural components, has proven to be computationally efficient for solving aerospace structures problems. The external aerodynamics of aerospace vehicles is dominated by field discontinuities such as shock waves and flow separations. FD computational methods

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have proven to be efficient for solving such flow problems.

In this paper, different types of fluid–structure interfaces and their pros and cons will be discussed. The discussions in the paper mainly deal with coupling of solutions from the ENS flow equations with different structural models by using the domain-decomposition approach suitable for both static and dynamic cases.

2. Domain decomposition approach

When simulating aeroelasticity with coupled procedures, it is common to deal with fluid equations in Eulerian reference system and structural equations in a Lagrangian system. The structural system is physically much stiffer than the fluid system, and the numerical matrices associated with structures are orders of magnitude stiffer than those associated with fluids. Therefore, it is numerically inefficient or even impossible to solve both systems using a single numerical scheme (see section on sub-structures in Ref. [5]).

Guruswamy and Yang [6] presented a numerical approach to solve this problem for two-dimensional airfoils by independently modeling fluids using the FD-based transonic small-perturbation (TSP) equations and structures using FE equations. The solutions were coupled only at the boundary interfaces between fluids and structures. The coupling of solutions at boundaries can be done either explicitly or implicitly. This domain-decomposition approach allows one to take full advantage of state-of-the-art numerical procedures for individual disciplines. This coupling procedure has been extended to three-dimensional problems and incorporated in several advanced aeroelastic codes such as XTRAN3S [7], ATRAN3S [8] and CAP-TSD [9] based on the TSP theory. It was also demonstrated that the same technique could be extended to model the fluids with the ENS equations on moving grids [10–12]. The coupled fluid structure analysis procedure using domain-decomposition approach is described in Section 3.

3. Coupled fluid structure analysis

Fig. 1 illustrates a time-accurate coupled fluid-structure aeroelastic analysis process. It is step-by-step time-integration procedure. Fluid and structural solutions are independently computed and the information is passed between them at common boundaries. At every time step the pressure data (C_p) from CFD are mapped on to structural grid points and force vector $\{Z\}$ is computed. Using Z , the structural displacements are computed from CSD analysis. Then deflections are mapped onto fluid grids that move accordingly. The

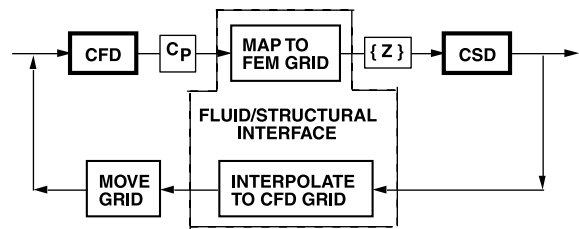


Fig. 1. Coupled fluid structural analysis.

interface techniques depend on the type of structural modeling.

Fluids and structural domains can be modeled at various levels of complexity both in physics and geometry. For design, aerodynamic data may be used at several levels of fidelity starting from the low-fidelity look-up tables to the high fidelity Navier–Stokes solutions. Similarly for structures, the data can be obtained starting from the low fidelity assumed shape functions to detailed three-dimensional FEs. As the fidelity of modeling increases, it becomes more difficult to handle complex geometry. Fig. 2 illustrates the typical levels of modeling complexities involved both for fluids and structures. Interfacing techniques depend on the levels of fidelity in both fluids and structures. In this paper, interfacing techniques between the ENS solutions and various levels of structural models are discussed.

Maintaining accuracy while transferring data between fluids and structures is important in order to obtain correct aeroelastic results. This can be accomplished with engineering accuracy by interpolations if fine grids are used both for structures and fluids. In these cases, the structural planform needs to be the same as the aerodynamic planform. Accuracy can be improved by

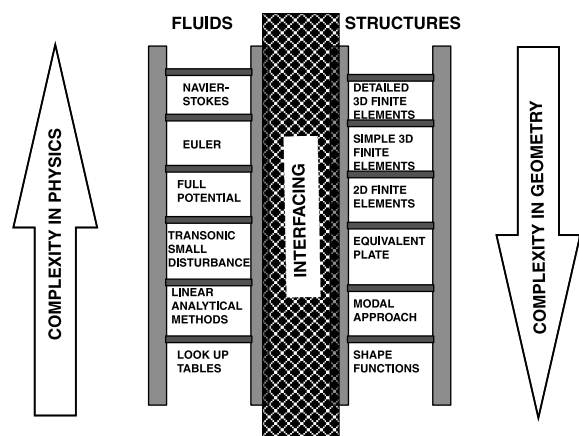


Fig. 2. Varying levels of fidelity in modeling for fluids and structures.

using higher-order interpolations. Often the structural mesh is either irregular, coarse or incomplete with respect to the fluid grid. Still, accurate interfacing can be achieved by balancing the loads and moments between fluid grids and structural nodes by using the consistent load approach [4,5] based on conservation of work. In this paper, consistent load, virtual-surface (VS) and moment-conservation approaches are presented for beam, plate, and wing box FE structural models, respectively.

4. Modal analysis

Modal representation of structures, based on the Raleigh–Ritz approach, is common in aeroelasticity [13]. The modal approach can compute accurate responses if an adequate number of modes are selected. The number of structural equations required in coupled calculations using the modal approach are typically an order of magnitude less than when direct FEM is used. Both static and dynamic responses can be accurately computed by using the modal approach in order to predict aeroelastic phenomena such as flutter [10].

The modal data for any configuration can be extracted by either ground vibration tests or FE analysis. Either approach can obtain modal data on a well-organized grid system. For example, the modal data can be obtained for simple plate-type wings on a modal grid for which grid lines run parallel to stream lines and along constant percentage chord lines as shown in Fig. 3. In the same figure, a wing-fitted FD surface grid is also shown. Ref. [14] gives a comprehensive summary of methods used to interface modal structures with fluids. In this paper interpolation and area-coordinate approaches that are suitable for the ENS equations are presented.

4.1. Interpolation approach

For modal structures, interpolation can be used to transfer the data between fluids and structures. The order of accuracy can be increased either by increasing the

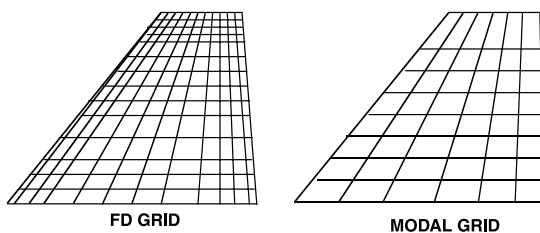


Fig. 3. Typical finite difference and FE grids suitable for bi-linear interpolation.

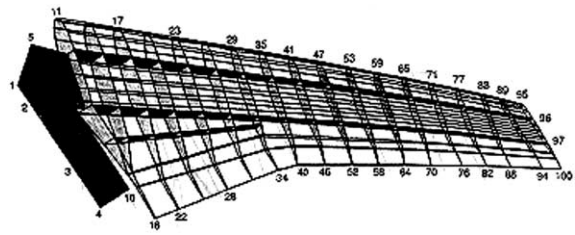


Fig. 4. ARW tested at Langley's wind tunnel.

grid densities or by increasing the order of interpolations. A procedure based on bi-linear interpolation is incorporated in the ENSAERO-WING code [12]. Since the same CFD and modal surface grids are used throughout the integration process, interpolation at every time step is avoided by mapping the modal data to the CFD surface grid data in the pre-processing stage. During integration, all structural computations are performed directly on the CFD surface grid which adds to the numerical efficiency of ENSAERO. For example, aeroelastic cases using ENSAERO run at 440 MFLOPS on a single Cray C-90 processor which is almost same as running steady state cases over rigid configurations.

Using the modal approach, static and dynamic responses are computed for several wings. One such wing is the advanced research wing (ARW) shown in Fig. 4 that was tested at the NASA Langley Research Center [15]. This wing was built as an aeroelastic wing with skin-spar-rib construction. Modal data were extracted on a 40×30 grid. Static deflections were computed using bi-linear interpolation and different CFD surface grid densities. It was found that a CFD surface grid adequate to resolve flow characteristics was also adequate for fluid/structure interpolation. Fig. 5 shows the static pressures and deflections computed for the ARW where CFD surface grids points 3500 and 4500 were used. The differences in results for the two grid sizes were negligible [16].

Based on the interpolation approach static aeroelastic deflections are computed for a wing-body configuration using ENS3DE [17] code that uses structured patched grid technology and the results are validated with wind-tunnel data.

4.2. Area coordinate approach

Quite often the modal grid is same as the original FEM grid which can be quite irregular. In that case, the best approach is to fit a triangular mesh to the irregular FEM mesh. Fig. 6 illustrates such a process. In the advanced version of ENSAERO a three-dimensional interpolation scheme between an unstructured triangular modal grid and structured CFD surface is incorporated.

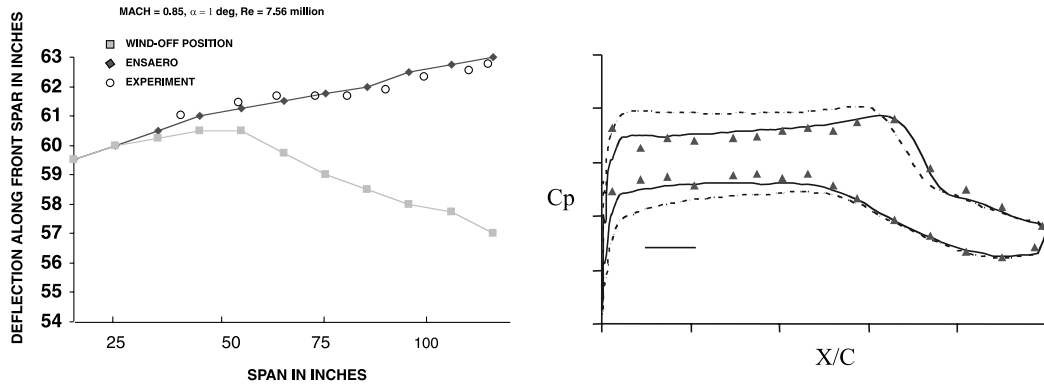


Fig. 5. Validation of fluid/structural interface for modal approach.

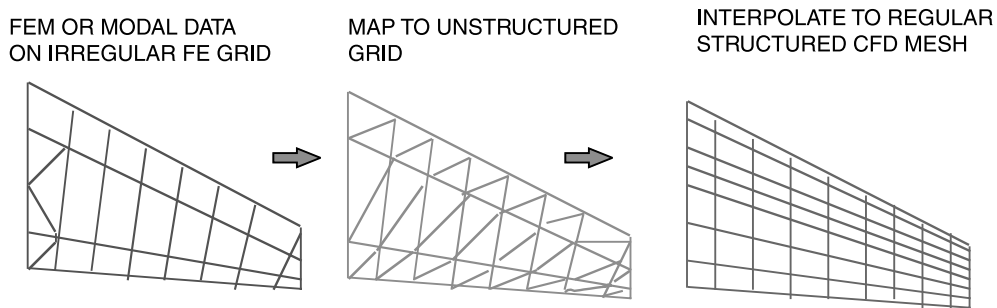


Fig. 6. Procedure to handle irregular modal grid.

In this process, fluid grid points are identified with respect to modal structural nodes i, j and k as shown in Fig. 7. The force at a fluid grid point is computed using the control area associated with that point. From the total force F at a given fluid point, the structural forces F_i, F_j and F_k are computed using

$$F_i = \frac{Fa}{(a+b+c)}; \quad F_j = \frac{Fb}{(a+b+c)}; \quad F_k = \frac{Fc}{(a+b+c)} \quad (1)$$

and

$$F = F_i + F_j + F_k$$

where a, b , and c are areas of the subtriangles in Fig. 7. This procedure is repeated for all fluid grid points.

The displacements w_i, w_j , and w_k from structural analysis, are interpolated to the fluid grid point using

$$d = \frac{w_i a + w_j b + w_k c}{(a+b+c)} \quad (2)$$

This approach is successfully used to interface data between NASTD Navier–Stokes code and NASTRAN for aeroelastic computations about F/A-18 stabilator and results are shown in Fig. 8 [18].

5. Beam structures

Transport-type configurations with high aspect ratio wings are commonly modeled by beam FEs. In this case,

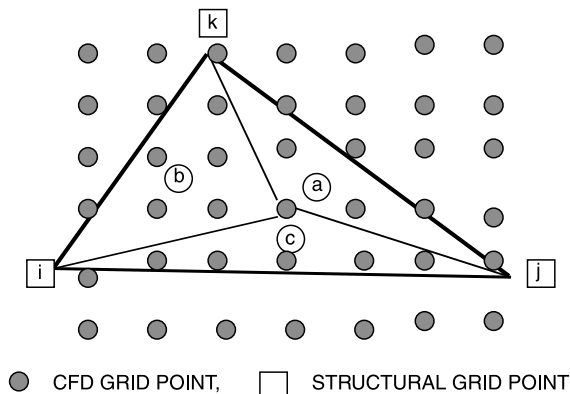


Fig. 7. Interpolation based on triangular area coordinates.

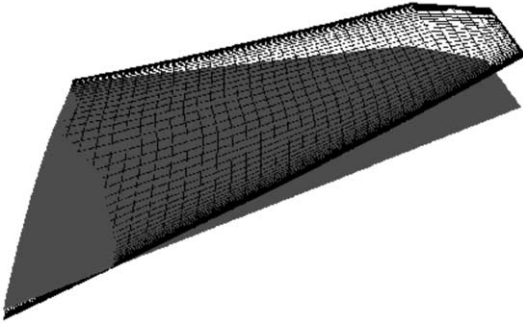


Fig. 8. Initial and final deflected position (magnified 10X) of F/A-18 stabilator.

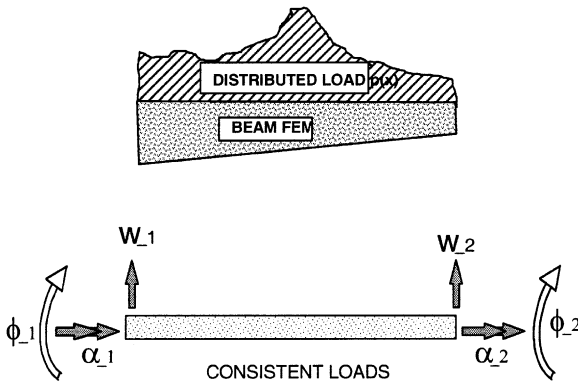


Fig. 9. Beam FEM with consistent loads.

the sectional lifts and moments are computed along the elastic axis of the body and wing. From the distributed load $p(x)$, the work-equivalent loads corresponding to nodal displacements w (transverse displacement), a (twist), F (bending) as illustrated in Fig. 9, are computed using the FE approach [4].

The unknown work-equivalent or consistent nodal loads are computed by equating the work done by nodal loads through nodal displacements to the work done by distributed load through distributed displacements. Using the FEM approach the consistent load vector can be computed using

$$F_i = \int p(x) f_i(x) dx \quad 0 < x < 1 \quad (3)$$

where i denotes the degree of freedom (DOF), $p(x)$ is the distributed load and $f_i(x)$ is the i th shape function (see Chapter 5, Section 4 of Ref. [4] for more details). Using this approach, computations were made on a wing-body configuration and results are presented in Ref. [19]. In Ref. [20], results are presented for the ARW [15] wing

using beam elements. Computed results compare well with the experimental data [15].

6. Plate/shell FEs

Some aerospace configurations can be modeled using plate and shell elements. Typical examples are fighter aircraft and launch vehicles. Quite often wind tunnel models are built out of solid material and can be modeled using plate/shell elements. In this case, both simple interpolation approaches that balance loads as well as the higher order interface techniques that conserve the work done by aerodynamic forces can be used. Such approaches were developed earlier in the context of linear aerodynamics and are reported in Refs. [21–25].

In any interpolation approach it is necessary to identify relative positions of the fluid and structural grid points. When the data are well organized with similar patterns both for fluids and structures as shown in Fig. 3, a simple search algorithm is adequate to find relative positions and interpolate the data. However, such approaches fail when geometry becomes more complex than a wing, say, even for a wing body configuration. In such cases it is difficult to have a consistent approach for both fluids and structural data.

6.1. Node-to-element approach

A wing-body configuration can be modeled by shell/plate elements such as the quadrilateral element shown in Fig. 9. The FE grids are generated using iso-parametric elements. For example, (x, y) coordinates the quadrilateral element shown in Fig. 10 can be represented by

$$\begin{aligned} x &= \sum N_i(\xi, \eta) x_i \quad 1 < i < 4 \\ y &= \sum N_i(\xi, \eta) y_i \quad 1 < i < 4 \end{aligned} \quad (4)$$

And the planar displacements u and v are expressed as

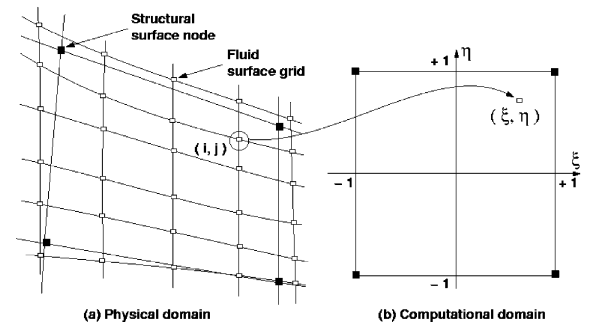


Fig. 10. Isoparametric quadrilateral shell/element.

$$\begin{aligned} u &= \sum N_i(\xi, \eta) u_i \quad 1 < i < 4 \\ v &= \sum N_i(\xi, \eta) v_i \quad 1 < i < 4 \end{aligned} \quad (5)$$

where

$$\begin{aligned} N_1(\xi, \eta) &= 1/4(1 - \xi)(1 - \eta) \\ N_2(\xi, \eta) &= 1/4(1 + \xi)(1 - \eta) \\ N_3(\xi, \eta) &= 1/4(1 + \xi)(1 + \eta) \\ N_4(\xi, \eta) &= 1/4(1 - \xi)(1 + \eta) \end{aligned} \quad (6)$$

The coordinate locations (x, y) of fluid-surface grid points can be defined with respect to the FE vertices $(i, j, k \text{ and } l)$ on the surface. For known (x, y) , the equivalent (ξ, η) values in the rectangular mapped domain can be computed using the numerical inverse mapping technique developed by Murti and Valliappan [25]. The aerodynamic force at a fluid grid point (x, y) location can be proportionately distributed to structural nodes using (ξ, η) values. Also the deformation obtained by FEM at nodes can be interpolated to fluid grid points using Eq. (5). In addition, linear extrapolation can be used to compute the deformation at the points of the fluid-surface grid which are not on the surface of the structure.

This approach is successfully applied for a wing-body configuration in Ref. [26]. An alternative approach based on the boundary element concept is presented in Ref. [27] and results are demonstrated for a blended wing-body configuration. In Ref. [28] a bi-cubic surface spline approach is suggested to transfer data between fluids and structures. In Ref. [29] an interface method to map aerodynamic forces computed on a wing surface to flat plate type structures is presented by accounting for the force transformations due to airfoil section thickness. Such methods are useful when moderately thick wings are modeled using equivalent plates.

6.2. Virtual surface method

One of the main efforts after selecting the FE model for the structure entails computing the global force vector $\{Z\}$ of Fig. 1 which is computed by solving the ENS equations at given time, t . First, the pressures are computed at all surface grid points. The forces corresponding to nodal DOF can be accurately computed using the FE nodal fluid-structural interfaces as discussed in the following section.

In aeroelastic analysis, it is necessary to represent equivalent aerodynamic loads at the structural nodal points and to represent deformed structural configurations at the aerodynamic grid points. In the domain decomposition approach, coupling between the fluid and structural domains is achieved by exchanging the boundary data such as aerodynamic pressures and structural deflections at each time step.

The previous interpolation and area coordinate approaches discussed in Sections 4.1 and 4.2 are also known as the lumped load (LL) approach from a structural analyst perspective. In the LL approach, the force acting on each element of the structural mesh is first calculated, and then the element nodal force vector is obtained by equally distributing the total force. The global force vector is obtained by assembling the nodal force vectors of each element. In addition, the deformed configuration of the CFD grid at the surface is obtained by interpolating nodal displacements at the FE nodes. This approach does not conserve the work done by the aerodynamic forces and needs fine grids for both fluids and structures to give accurate results.

In the VS approach, a mapping matrix developed by Appa [21] and Appa et al. [23] is selected to accurately exchange data between the fluid and structural interface boundaries. The reason for selecting the method of Appa and colleagues is that the mapping matrix is general enough to accommodate changes in fluid and structural models easily. In addition, this approach conserves the work done by aerodynamic forces when obtaining the global nodal force vector.

The VS method introduces a VS between the CFD surface grid and the FE mesh for the wing. This VS is discretized by a number of FEs, which are not necessarily the same elements used in the structural surface modeling. By forcing the deformed VS to pass through the given data points of the deformed structure, a mapping matrix relating displacements at structural and aerodynamic grid points is derived as

$$[T] = [\Psi_a](\delta^{-1}[K] + [\Psi_s]^T[\Psi_s])^{-1} \quad (7)$$

where $[K]$ is the free-free stiffness of the VS, Ψ_s is a displacement mapping from VS to structural grids, Ψ_a displacement mapping from VS to aerodynamic grids, and δ is the penalty parameter (see Refs. [21,23] for more details).

Then, the displacement vector at the aerodynamic grid, q_a , can be expressed in terms of the displacement vector at the structural nodal points, q_s as

$$q_a = [T]\{q_s\} \quad (8)$$

From the principle of virtual work, the nodal force vector, $\{Z_s\}$, can be obtained as

$$\{Z_s\} = [T]^T\{Z_a\} \quad (9)$$

where $\{Z_a\}$ is the force vector on the aerodynamic grids. This method is illustrated in Fig. 11.

To demonstrate aeroelastic computations, a typical fighter-type wing is selected. For this wing transonic flutter data are available from wind tunnel tests [30]. In this computation, the flow field is discretized for Euler-type solutions using a C-H grid topology of size

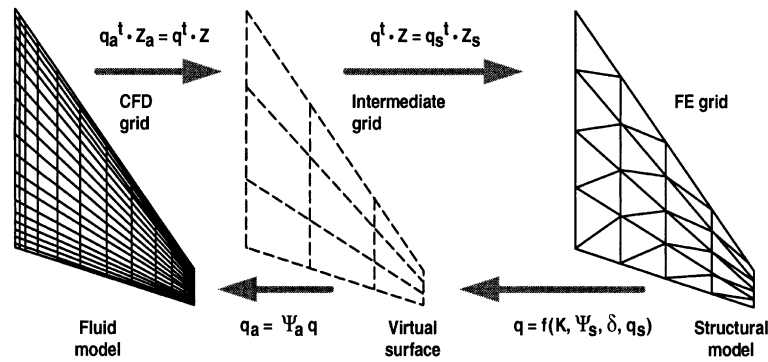


Fig. 11. Fluid–structure interface using transfer functions.

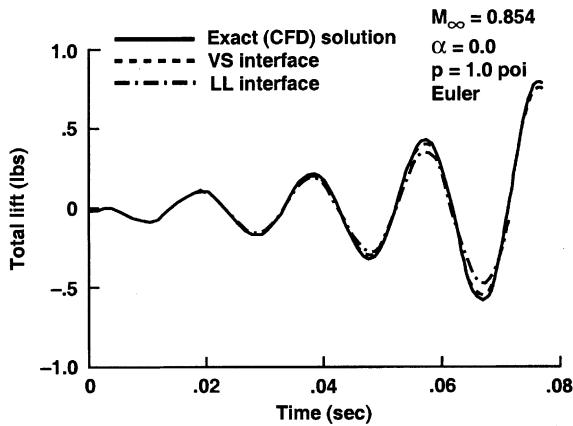
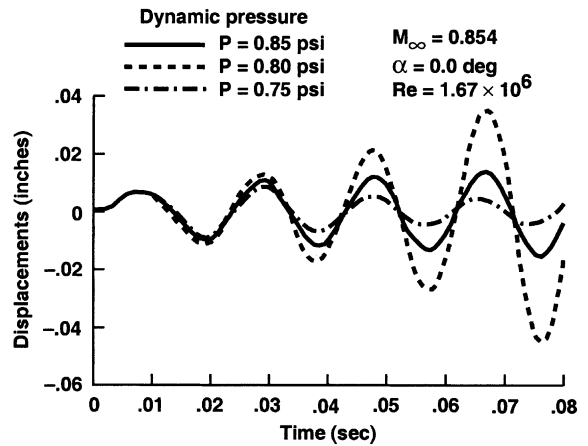


Fig. 12. Comparison of total lift responses.

Fig. 13. Computation of flutter boundary (expt. dynamic pressure $p = 0.91$ psi).

$151 \times 30 \times 35$. The wing structure was modeled using 100 ANS4 elements. Ten elements each were assigned along the chordwise and spanwise directions. To discretize the VS, four-noded isoparametric elements are used.

The accuracy of the results can depend on the type of interfaces between fluids and structures. The simple LL and the more accurate VS interfaces are compared to each other. Results are shown in Fig. 12. For a given dynamic pressure of 1.0 lbs/in.^2 and initial acceleration of $1.0 \times 10^5 \text{ in./s}$, the time history of total lift on the wing is presented in Fig. 12. The total lift obtained by integrating the pressure coefficients at CFD grid points is also shown in the figure.

The total lift using CFD grid points is more accurate than those from the VS and LL methods. Both VS and LL approaches obtain the total lift by summing the forces at the FE nodal points, which was transformed from the pressure coefficients through interfaces. The VS approach transfers pressure data more accurately than

the LL approach. The LL approach shows that the response around peaks deviates from the CFD solution. For this case the LL approach shows reasonable agreement with the VS approach.

Based on the VS approach, the flutter dynamic pressure of the wing was computed using response analysis as shown in Fig. 13. The computed flutter dynamic pressure is 0.85 lbs/in.^2 compared to the wind tunnel measurement of 0.91 lbs/in.^2 . Given the uncertainties in the model a 6.5% error in predicting the transonic flutter boundary is reasonable. Based on the results shown in Fig. 12 the LL approach can also give reasonably accurate results.

Among all methods presented to date the VS approach has strong potential for general applications dealing with more complex geometries and complete equations. Application of this approach for wing-body configurations by using the Navier–Stokes equations is demonstrated in Ref. [31] for moderately separated

turbulent flows. The accuracy of VS approach can be further verified using tests described in Refs. [32,33].

7. Wing-box FEM model

Most transport aircraft are built using spars, ribs, skin and bulk-head type components. For the best accuracy, it is necessary to model them directly using FEs. This will avoid truncation errors caused by other simpler approaches such as the modal approach. Fig. 14 illustrates the planform and internal model of a typical spar-rib-skin construction also commonly known as a “wing-box model”. Only the components between spars are normally considered for structural analysis. The rest of the portions near the leading and trailing edges are treated as dead weight.

However, from an aerodynamic point of view the complete surface needs to be considered. This leads to a mismatch in the surface definitions between fluids and structures. An accurate procedure that accounts for twist correction (compensation) was developed in

Ref. [34] and is briefly described here. Lumping forces at structural nodes balances the total load but not the bending and twist moments. The moments can be conserved by computing the structural loads with the following system of equations with respect to f_i , $i = 1-3$.

$$\sum_{i=1}^3 f_i = f_a; \quad \sum_{i=1}^3 f_i x_i = f_a x_a; \quad \sum_{i=1}^3 f_i y_i = f_a y_a \quad (10)$$

where ‘a’ denotes aerodynamic grid point, ‘i’ represents three non-colinear structural nodes and ‘x’ and ‘y’ are coordinates distances from reference axes.

The deflections computed at FEM nodes are transferred to the fluid grid points using transformation functions. It is assumed that the wing is chordwise rigid. The aeroelastic deflections are reduced to a set of leading edge deflections and rotations about the leading edge as shown in Fig. 15.

The present method was demonstrated in Ref. [34] for a sample wing shown in Fig. 16. Computations were performed at $M = 0.90$ and $\alpha = 4^\circ$ using a NACA 64A10 airfoil section. The fluid is modeled using a C-H grid of size $151 \times 25 \times 35$. The differences in span-wise displacements using simple load-lumping and moment conservation are shown in Fig. 17. Because of significant differences between fluid and structural grid sizes, corrections for moments strongly influence the results.

A procedure that combines the consistent-load approach presented for plate elements in Section 6.1 and twist-correction approach presented in this section is proposed in Ref. [35] for interfacing data between fluids and structures.

8. Detailed FE model

Complete FE models of structures are needed for final aircraft design. In this case the interior of the FEM grid can be very irregular as shown in Fig. 18. Surface elements may consists of either triangular or quadrilat-

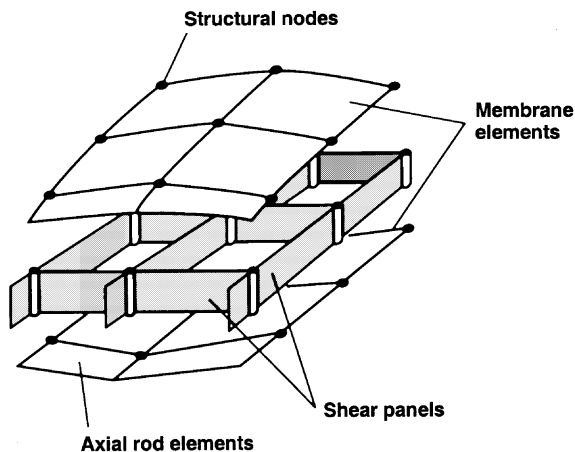


Fig. 14. Typical wing-box FEM model.

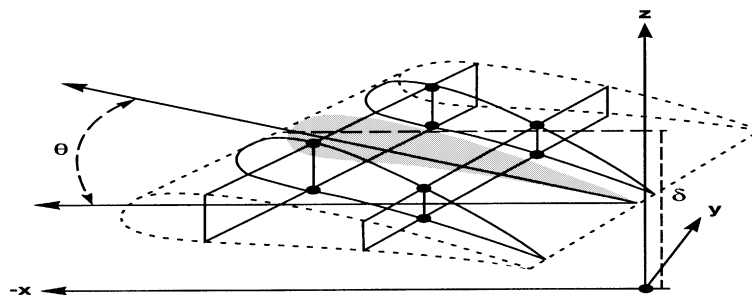


Fig. 15. Transfer of deflection to aerodynamic grid points.

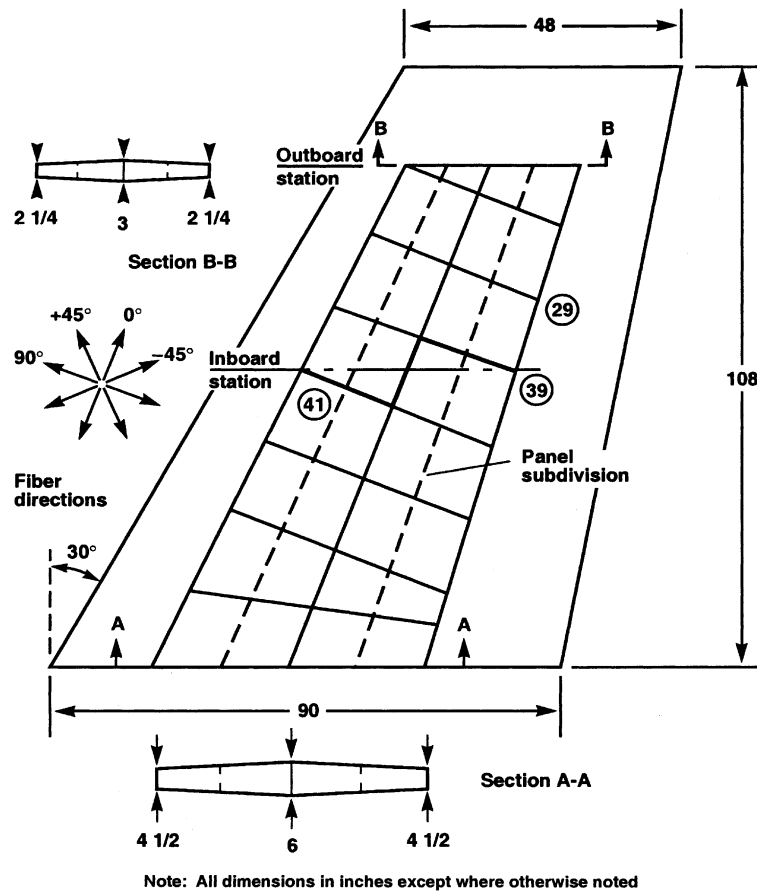


Fig. 16. Aerodynamic planform and primary structural arrangements of a wing.

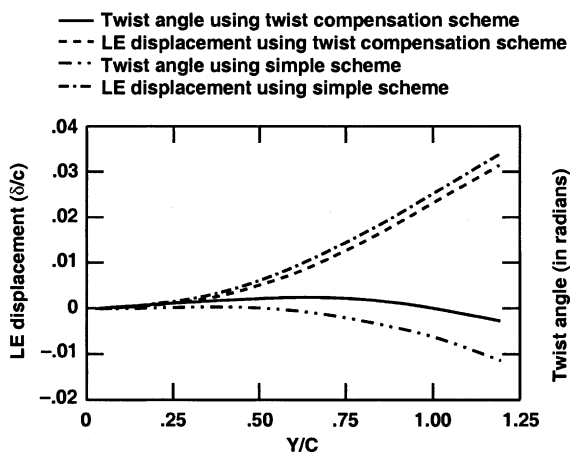


Fig. 17. Effect of moment correction on wing deformation.

eral skin elements. Sometimes, near the root, solid elements may be used. For such configurations the first step is to create a triangular surface mesh that has common

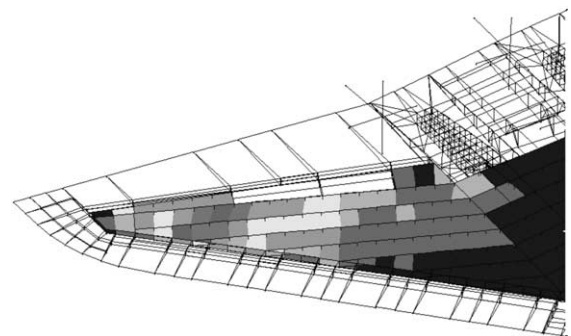


Fig. 18. Typical FE model of a real wing.

nodes with the surface FE nodal points. Next, either the area coordinate method (Fig. 8) or the more accurate VS method can be used to transfer deformations and loads. A three-dimensional area coordinate type of interface is used to directly couple data between NASTRAN and ENSAERO [36]. Figs. 19 and 20 illustrate results for a wing-body configuration.

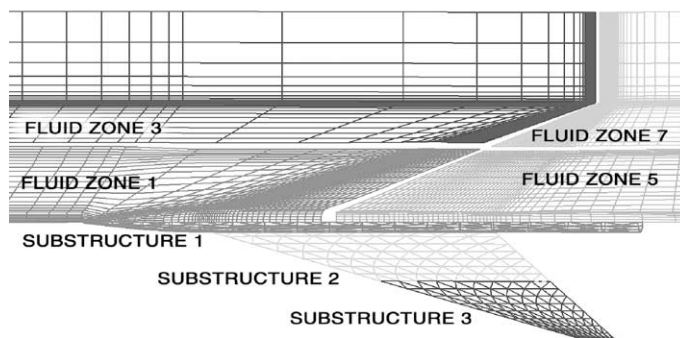


Fig. 19. Multi-zone fluids and structural grid layout.

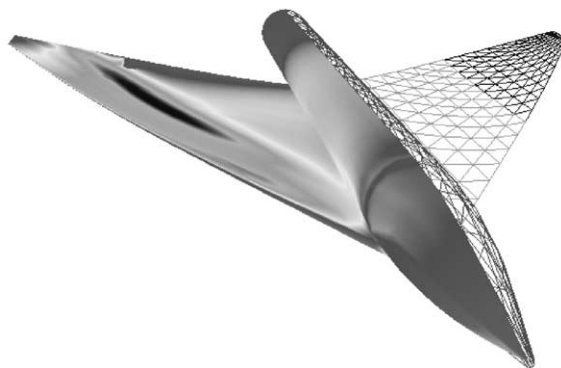


Fig. 20. Static aeroelastic solution using ENSAERO and NASTRAN.

All coupled methods described so far in the paper dealt with using structured grids for fluids and unstructured grids for structures. Efficient fluid/structure coupled method based on unstructured grids both for fluids and structures is implemented in computer code ADINA [37].

9. Conclusions

Several methods that are suitable to interface the Navier–Stokes/Euler equations based fluids solutions with the FEM/modal based structural analysis is presented. The interface depends mainly on the type of structural modeling. The accuracy of the interfacing can be increased either by using fine surface structural meshes similar to fluids grids or using higher order interpolations such as splines. Interfacing approaches that conserve work show improved accuracy over the lumped load approach.

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